BACKWARDATION IN OIL FUTURES MARKETS: THEORY AND EMPIRICAL EVIDENCE

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Abstract

Oil futures markets frequently exhibit backwardation whereby more distant oil futures prices are below the current spot price. This is inconsistent with Hotelling's theory that the net price of an exhaustible resource rises over time at the interest rate. We characterize an oil well as a call option and show that backwardation is necessary to induce production. Production is shown to be non-increasing in the riskiness of future prices. The empirical analysis indicates that U.S. oil production is directly related to the backwardation and inversely related to implied volatility. Backwardation is positively related to implied volatility and to the at-the-money put option price.

1 Introduction

Oil futures markets frequently exhibit strong backwardation whereby more distant oil futures prices are below the current spot price. Even more frequently they exhibit weak backwardation whereby *discounted* futures price are lower than the current spot price. During the period between February 1984 and April 1992 the nine months futures price was strongly backwardated 77% of the time and weakly backwardated 94% of the time.[1] The existence and persistence of backwardation in oil futures markets appears to be inconsistent with Hotelling's (1931) theory that the net price (price less marginal extraction cost) of an exhaustible resource should rise over time at the rate of interest.[2] This paper presents a theory for backwardation based on the characterization of oil in the ground as a call option. We argue that it is not the presence of weak backwardation 94% of the time but rather its absence 6% of the time which should be viewed as puzzling.

While the characterization of natural resources as options was understood long ago,[3] its role in the formation of backwardation has not been previously examined. This paper uses the option aspect of oil in the ground to explain why the oil market should exhibit weak backwardation, and why it may exhibit strong backwardation. Backwardation stems from the tradeoff between exercising the option (i.e., producing the oil) and keeping it alive (i.e., leaving the oil in the ground). It seems that whenever the market is weakly backwardated, producers will want to extract all of their oil, sell it in the spot market and use the proceeds to purchase future oil at a lower price.

[1] Table 1 presents summary statistics of weak and strong backwardation for the West Texas Intermediate (WTI) crude oil contracts (traded on the NYMEX). It also shows the fraction of time futures prices were weakly or strongly backwardated. Figure 1 shows the term structure of futures prices on a typical day of strong backwardation. Figures 2 presents the futures price of the nearest to maturity contract and the weak backwardation for the nine months contract.

[2] Elaborated discussion follows in section 2.

[3] See, for instance, Tourinho (1979).

However, if discounted futures price are higher than its spot price, producers will choose to defer production. In this case they can extract the oil at a future date but do not have to do so. They are protected against adverse price realization. Therefore, in order to induce them to extract some oil now weak backwardation is required. If the uncertainty about futures prices is substantial, strong backwardation will be necessary in order to induce current production.

Our initial analysis is based on a two period economy where the reserves of oil are owned by a continuum of price taking oil producers with heterogeneous extraction costs. There are spot markets for oil, as well as financial markets for futures and options contracts on oil. Each producer has to decide whether or not to exploit his reserves in the first period. The price of oil in the second period is stochastic in the presence of a random demand shock. Hence, oil reserves which are not extracted in the first period will be extracted in the second period only if the realized price is higher than the extraction cost. In this framework we prove the existence and uniqueness of equilibrium in the market and present sufficient conditions for an inner equilibrium. We then demonstrate that in the case of an inner equilibrium the market always exhibits weak backwardation and may exhibit strong backwardation. The weak backwardation is equal to the value of a put option with an exercise price equaling the extraction cost of the marginal producer. This result is very intuitive if we view backwardation as an insurance premium borne by the oil producer who is protected against adverse price realizations in the future. If the put option value is sufficiently large (for instance, when the volatility of prices is high) strong backwardation appears. If the demand shock is degenerate, the put option value is zero and Hotelling's result (price rising at the rate of interest) is obtained.

An interesting outcome of our model is that equilibrium production decreases (or remains unchanged) as the demand shock becomes "riskier" in the sense of mean preserving spread. This outcome follows from the appreciation of the option element of oil in the ground which induces producers to defer extraction.

Next, we show that our main result carries over to a multiperiod framework. More specifically, we show that that in any inner equilibrium the market is weakly backwardated with respect to any future time t. We also show that weak backwardation for time t is bounded from below by the value of a European futures put option with a strike equal to the time t extraction cost of the marginal producer.

We then proceed to empirically test some predictions of the model. For our tests we use U.S. production and reserves data as well as WTI futures and option prices. We first examine how production relates to the expected volatility of future prices and to the level of weak backwardation. We find that oil production rate (production as a fraction of reserves) for non-regulated states has a significant negative relationship with implied volatility from call option prices. It has a positive but non-significant relationship with the level of weak backwardation.

Next, we test the relationship of weak backwardation with the put option of the marginal producer. Since the extraction cost of the marginal producer is unobservable we regress weak backwardation on the at-the-money futures put option price. We find a highly significant positive relationship. We notice that the put price coefficient is less than one. This is consistent with the extraction cost of the marginal producer (the strike price suggested by the model) being lower than the futures price (the strike for the at-the-money put option used in the regression). We also notice that the put price price coefficient increases with maturity towards one. This can be explained by the fact that the futures price decreases with extraction cost increases with maturity, and hence the difference in value between the put option suggested by the model and the at-the-money put option declines with maturity.

The put options suggested by the model are not observable. Nevertheless, these options as well as the observed options are positively related to the expected volatility of oil prices. Hence, as an alternative test we regress weak backwardation on implied volatility from the traded options. The relationship is shown to be positive and significant and provides an additional support for the above result.

The paper proceeds as follows: In section 2 prior literature on the price behavior of exhaustible resources is discussed. In section 3 we present the model and derive the results for equilibrium production, prices and backwardation in the market. Empirical tests of the model's predictions are presented in section 4. A summary and some concluding remarks are included in section 5.

2 Literature critique

In his seminal paper on exhaustible resources, Hotelling (1931) derived the well known "Hotelling's principle" under the conditions of perfect competition and certainty. This principle states that the net price of the resource,[4] say crude oil, should rise over time at the rate of interest. The result is very intuitive. In case of an inner equilibrium in the market, each one of the producers is indifferent between present and future exploitation of the oil. This implies that the discounted profits are equal for all periods; otherwise, if the profits rise by less than the interest rate, all reserves will be produced immediately, and in the opposite case — no current production will occur. Assuming in addition that the extraction cost per unit is proportional to the price of oil,[5] the above result implies that the price of oil itself appreciates at the rate of interest. However, if we assume that the extraction cost per unit has a fixed component, the result is not preserved. Whenever the fixed extraction cost rises by less than the interest rate, so does the price of oil.

Under certainty the future spot prices are known and are equal to current futures prices. Hence, the market exhibits weak backwardation whenever the fixed component in the per-unit extraction cost rises by less than the interest rate. However, a necessary (but not sufficient) condition for strong backwardation is extraction costs *decreasing* fast enough over time. As shown in section 3, the presence of uncertainty can explain strong backwardation even if the extraction costs increase over time.

Introduction of uncertainty by itself does not necessarily lead to backwardation in the market. Hotelling's principle is preserved under uncertainty if we maintain the assumption that extraction cost per unit is proportional to the price of oil. However, uncertainty coupled with a fixed component in the per-unit extraction cost results in the

[4]Net price, as used by Hotelling, means the price minus the extraction cost per unit, assuming constant returns to scale.

[5]Extraction cost would be proportional to the price of oil if oil by itself were the only productive input in extraction.

reserves of oil having a call option characteristic. We show later that due to this feature the market will exhibit weak backwardation and may exhibit strong backwardation. This is true even if extraction cost rises over time at the rate of interest, which would imply a situation of no backwardation in the Hotelling world. In general, the source of uncertainty can vary : it can be stochastic demand where it is possible to have low realization under which no oil is extracted; it can be stochastic extraction cost or stochastic reserves (where the extraction cost is a function of reserves) with the possibility of high cost realization.

One explanation for backwardation that has been suggested in the literature is that of "convenience yield" (See Kaldor (1939), Working (1948)). Convenience yield, as defined by Brennan and Schwartz (1985), is "*the flow of services that accrues to an owner of the physical commodity but not to an owner of a contract for future delivery of the commodity*". Brennan and Schwartz (1985) in a one factor model, as well as Gibson and Schwartz (1990) in a two factor model, associate backwardation with convenience yield. However, in contrast with our model, these models assume that the convenience yield is exogenous and therefore do not analyze the formation of backwardation in the oil market. As convenience yield depends to the notion of storage, we note that unlike many other exhaustible resources, crude oil is almost a non-storable commodity. The storage costs of crude oil as a percentage of its value are extremely high. In addition, storage capacity for oil above the ground is very limited. In fact, refiners' oil inventories are almost never held for more than thirty days and in most cases the oil is processed as soon as it reaches the refinery. In light of this, we find the convenience yield explanation unsatisfactory in the context of the crude oil market. We claim in this paper that the answer to the backwardation puzzle lies in the oil producers' extraction problem and its related option aspect.

Sundaresan (1984) analyzes the producers' extraction problem within an equilibrium framework. He considers monopolistic as well as price taking producers. In contrast with our main result, his model suggests that discounted futures prices are always *higher* than the spot price. This result, however, is driven by his assumption of zero extraction cost. Under this assumption the value of oil in the ground is simply equal to its intrinsic value and the additional option element is lost.

Pindyck (1980) also investigates the price behavior of exhaustible resources under uncertainty. He allows for both demand and reserves uncertainty and does not assume zero extraction cost. Surprisingly, he finds that the expected future prices obey Hotelling's principle. This outcome, however, is driven by the fact that producers in his model employ a non-optimal stopping rule. Pindyck assumes that production permanently stops as soon as the extraction cost exceeds the price. When this occurs, the reserves which are not yet extracted are lost and the producers do not have the option to resume production in the future.

In this context, it is worthwhile mentioning that for certain oil wells a complete cessation of production can cause damage to the total recovery. In the extreme, as in the case of stripper wells, an oil well that is closed cannot be reopened at any cost. Therefore, even if the spot price of oil is low and does not reach the extraction cost, it may be worth maintaining a minimal level of production in order not to lose the option of producing in the future. This feature can potentially explain the short periods of time when the market did not exhibit weak backwardation.

3 The equilibrium model

In this section we present the model and characterize the equilibrium in the market. In a two period framework it is shown that the market is always weakly backwardated in an inner equilibrium. The weak backwardation is equal to the value of a put option with a strike equaling the extraction cost of the marginal producer. If the value of the put is sufficiently large strong backwardation emerges. Oil production is shown to be non-increasing in the riskiness of future prices. The existence of weak backwardation in an inner equilibrium is generalized to a multiperiod framework, and a lower bound for the backwardation is provided.

3.1 Producers' supply

Consider a two period economy with finite reserves of oil, Q. Q_0 of the reserves are available at time 0, and Q_1 of the reserves are discovered at time 1.[6] There is a continuum of heterogeneous oil producers, each of whom owns an equal share of reserves. They are uniformly distributed with respect to their extraction cost from 0 to $\bar{x}$. Thus, denoting the aggregate production by q, the extraction technology (presented in Figure 3) is given by:

$$q = \frac{x}{\bar{x}} Q \ , \tag{1}$$

for $x \in [0, \bar{x}]$.

The extraction cost for each of the producers is assumed to increase by the interest rate from time 0 to time 1. Thus, the extraction cost at time 0 ranges from 0 to $e^{-r}\bar{x}$, where r is the interest rate. We choose to focus on this case since it would imply a situation of no backwardation under Hotelling's conditions. In this way we are able to emphasize the option effect as the source of backwardation.

[6]The addition of Q_1 is introduced in order to avoid the complications associated with zero supply at time 1. For this purpose Q_1 can be arbitrarily small. But even with $Q_1 = 0$ our results can be shown to hold.

Oil is traded in the spot markets in both periods. S_0 is the time 0 spot price. The future spot price, $\tilde{S}_1$, is a random variable as of time 0. Futures and option contracts are also traded. The futures contract is a cash settlement contract written on time 1 oil with the futures price F . The call and put options are available for any exercise price and their prices are denoted by C_k and P_k (respectively), where k is the exercise price.

Consider now an oil producer whose cost of extraction at time 1 is x . The net revenue for this producer from extracting one unit of oil at time 0 is given by $S_0 - e^{-r}x$. If the producer chooses not to extract, the oil is left for time 1 where its price, $\tilde{S}_1$, is random. Then, if that the price exceeds the producer's extraction cost, each unit of oil will earn a net revenue of $S_1 - x$. Otherwise, the oil will not be extracted and the revenue will be zero. Notice that the payoff of a unit of oil in the ground is exactly spanned by the traded call option with an exercise price of x . The value maximizing production policy is given by,

$$S_0 - e^{-r}x \; < \; C_x \quad \Rightarrow \quad \text{Leave oil in the ground} \tag{2}$$

$$S_0 - e^{-r}x \; = \; C_x \quad \Rightarrow \quad \text{Indifferent} \tag{3}$$

$$S_0 - e^{-r}x \; > \; C_x \quad \Rightarrow \quad \text{Extract oil now .} \tag{4}$$

As can be seen from (2)-(4), each one of the producers decides whether or not to extract his oil by comparing its net value above the ground (spot price minus extraction cost) with its value in the ground (the price of the call option). No producer will store oil above the ground, assuming that extraction costs do not rise by more than the interest rate.[7] This is the case since underground storage is costless (no physical storage cost) and provides a protection against adverse price realizations in the future (the call option feature). In the aggregate, the supply of oil at time 0 is a function of the spot

[7]Extraction costs rising by no more than the interest rate is a strong sufficient condition and obviously not necessary.

price and the other factors which determine the call prices schedule. It is given by,

$$q^s = \begin{cases} 0\ , & \text{if} \quad S_0 \leq e^{-r}F \\ \frac{x}{\bar{x}}Q_0\ , & \text{if} \quad S_0 - e^{-r}x = C_x \ \text{ for some } \ x \in (0, \bar{x}) \\ Q_0\ , & \text{if} \quad S_0 - e^{-r}\bar{x} \geq C_{\bar{x}} \end{cases} \tag{5}$$

The supply of oil is zero if $S_0 \leq e^{-r}F$. In this case even the zero cost producer does not produce since the value of his oil above the ground, S_0 , is lower than its value in the ground, C_0 (notice that $C_0 \equiv e^{-r}F$). All other producers, whose extraction costs are higher, do not produce as well since $S_0 - e^{-r}x - C_x$ is non-increasing in x.[8] The supply is Q_0 if the highest cost producer, and hence all other producers, decides to extract. This is the case if $S_0 - e^{-r}\bar{x} \geq C_{\bar{x}}$. In all other cases the aggregate supply at time 0 is between 0 and Q_0 , as described in (5). We obtain, therefore, that the supply of oil is a non-decreasing function of the spot price and a non-increasing function of any other variable which is positively related to the call price.

3.2 Consumers' demand

We turn now to a description of the demand for oil. Consumers are assumed to be risk neutral. We assume linear demand functions for the first and for the second periods. Uncertainty is introduced through a random parallel shock to the second period demand curve. The demand functions for time 0 and time 1 are,[9]

$$D_0 = a - bS_0 \tag{6}$$

and

$$D_1 = a + \tilde{\epsilon} - bS_1\ , \tag{7}$$

[8]See Lemma 1a, appendix A.

[9]This demand structure assumes no temporal substitution of consumption between the two periods. However, the result of backwardation in equilibrium is not driven by this assumption. Any structure of less than perfect temporal substitution will do in our model; we choose the extreme case for reasons of simplicity.

where a and b are positive. The shock, $\tilde{\epsilon}$, is assumed to have a zero mean so that the average demand at time 1 is the same as the demand at time 0. We rule out the trivial case where the first period demand curve lies entirely below the highest extraction costs; i.e., we require that $a/b > \bar{x}$.[10] Thus, the parameter space and the random shock space are given by:

$$\begin{aligned} \Theta &\equiv \{\theta \equiv (a, b, \bar{x}, r, Q_0, Q_1) \in I\!R^6_+ \backslash \{0\} \mid \frac{a}{b} > \bar{x}\} \\ \mathcal{E} &\equiv \{\tilde{\epsilon} \mid \tilde{\epsilon} \in \mathcal{L}^1 \text{ and } E(\tilde{\epsilon}) = 0\}. \end{aligned}$$

Convenience yield from storing oil above the ground can result from discreteness in oil shipments, cost of disruptions in the refining process and locational basis due to time lag and cost of transportation.[11] Abstracting from this, a necessary condition for consumers to store oil in the presence of physical storage cost is the spot price of oil being lower than its discounted futures price. In other words, consumers will not store oil unless $S_0 < e^{-r}F$.

3.3 Equilibrium and backwardation

The equilibrium production and prices are determined by the intersection of supply and demand.[12] The equilibrium is characterized by the marginal producer. We denote by $x^e(\tilde{\epsilon}, \theta)$ the extraction cost of this producer. $S^e_0(\tilde{\epsilon}, \theta)$ and $\tilde{S}^e_1(\tilde{\epsilon}, \theta)$ denote the equilibrium prices (the arguments will be suppressed henceforth). A unit of oil in the ground owned by the marginal producer pays off at time 1 $\max[\tilde{S}^e_1 - x^e, 0]$. Its value as of time 0,

[10] In this way we allow for the possibility that all initial reserves will be extracted at time 0.

[11] Storage of oil above ground is minimal and is associated with extremely high physical storage cost per unit value. Most of these small stocks of oil are in a state of transition (in pipelines, trucks or tankers) from oil fields to refineries. The rest is held on refinery sites due to discreteness of oil shipments and to allow for smooth refining process. Imported oil carried in tankers to the U.S. may have a speculative aspect to it as the tankers can change their speed of sailing or even their final destination in order to take advantage of locational basis.

[12] An illustration of these curves at time 1 is given in Figure 4.

which is now determined endogenously, is the same as the value of a call option with the exercise price of x^e , and is denoted by C_{x^e} . In the case of an inner equilibrium the marginal producer is just indifferent between extracting his oil at present and leaving it in the ground,

$$S_0^e - e^{-r}x^e = C_{x^e} \ . \tag{8}$$

Without further restrictions on the parameters and on the demand shock, boundary equilibria are possible as well.[13] First, consider the boundary equilibrium of no production at time 0. For the lowest cost producer (zero extraction cost) the value of oil in the ground as of time 0 is simply equal to the discounted futures price. If the spot price is lower than the discounted futures price (no weak backwardation) the zero cost producer will not produce. The other producers (who have higher extraction cost) will not extract their oil either.[14] Hence, no production occurs. Next, consider the other boundary equilibrium of full production. Consider the highest cost producer who has an extraction of $e^{-r}\bar{x}$ per unit at time 0. The value of one unit of oil in the ground for this producer is equal to the value of a call option with an exercise price of $\bar{x}$. If the net spot price of oil at time 0, $S_0^e - e^{-r}\bar{x}$, is higher than the call option value, $C_{\bar{x}}$, then the highest cost producer will extract the oil. All the other producers (who have lower extraction cost) will extract their oil as well. Hence, all reserves are depleted in the first period.

We now turn to the more interesting case, that of an inner equilibrium. We use the following notation: F^e denotes the equilibrium futures price of oil; C_{x^e} and P_{x^e} denote the prices of call and put options with the exercise price of x^e ; and finally, the equilibrium level of weak and strong backwardation are denoted by,

$$B_w^e \equiv S_0^e - e^{-r}F^e \tag{9}$$

$$B_s^e \equiv S_0^e - F^e \tag{10}$$

[13]Theorem A1 in appendix A provides necessary and sufficient conditions for boundary and for inner equilibria. The uniqueness of the equilibrium is proven there as well.

[14]This result follows from the monotonicity of $S_0(x) - e^{-r}x - C_x$ in x. See Lemma 1, appendix A.

We present now the following result with respect to the backwardation in equilibrium:

Theorem 1 In an inner equilibrium the market is weakly backwardated. The weak backwardation is equal to the value of a put option with an exercise price of x^e, the extraction cost of the marginal producer. Strong backwardation, if it exists, is equal to the put option value minus the interest factor of the equilibrium futures price. That is,

for all $(\tilde{\epsilon}, \theta) \in (\mathcal{E} \times \Theta)$ such that $q^e(\tilde{\epsilon}, \theta) \in (0, Q_0)$, we have

$$B_w^e = P_{x^e} \ , \tag{11}$$

$$\text{and} \quad B_s^e = P_{x^e} - (1 - e^{-r})F^e \ . \tag{12}$$

Proof As stated in equation (8), in case of an inner equilibrium the marginal producer is indifferent between producing his oil at time 0 or leaving it in the ground. Its value in the ground is the call option value for which the put-call parity holds:

$$C_{x^e} = e^{-r}(F^e - x^e) + P_{x^e} \ . \tag{13}$$

In this way the value of the reserves in the ground can be viewed as the sum of the guaranteed discounted net revenue at time 1, $F^e - x^e$, plus the "insurance premium" — the put option value. To obtain the results of the theorem just combine the equilibrium condition (equation (8)) with the put - call parity (equation (13)). □

Examining the results of theorem 1, we can clearly understand the conditions under which backwardation exists in equilibrium. Weak backwardation is equal to the put option value with exercise price of x^e . The put option value is always non-negative. In fact, it is strictly positive in an inner equilibrium, for if it is not the case there will be no production at time 0. As suggested by the second result, the market exhibits strong backwardation if the put option value is larger than $(1 - e^{-r})F^e$. This can be the case if the uncertainty about future prices is sufficiently large. In the case of a degenerate demand shock, (i.e. under certainty), all producers are indifferent between present and future production and the backwardation is zero. Hence, the futures price is equal to the spot price compounded by the interest rate, which is Hotelling's original result under certainty.

It is important to note that theorem 1 provides a necessary condition for the existence of an inner equilibrium in the market. In any event that the extraction cost increases by no more than the interest rate, a necessary condition for an inner equilibrium is weak backwardation in the market.[15] In this sense, the existence and persistence of backwardation in the oil market are not at all puzzling. It is the *absence* of backwardation, whenever it occurs, which should be viewed as the puzzling phenomenon. As explained in section 2, the occurrences of "negative backwardation" might be explained by the need to maintain a minimal level of production even when the price does not reach the extraction cost, in order to prevent a partial or total damage to the reserves.

We present now the following corollary concerning the storage of oil above ground:

Corollary 1 In any equilibrium there is no storage of oil above the ground.

Proof As suggested earlier (section 3.1), producers will never store oil above the ground when extraction costs rise by no more than the interest rate — underground storage is costless (no physical storage cost) and provides a protection against future adverse price realizations. With respect to the consumers (as suggested in section 3.2), a necessary condition for storing oil above the ground is the absence of backwardation (i.e., $S_0 < e^{-r}F$). But Theorem 1 implies that weak backwardation is necessary for production to take place. Hence, the is no storage of oil above the ground in equilibrium. □

We next examine the effect of increased uncertainty about future prices on equilib-

[15]But even with extraction cost rising faster than the interest rate we can get weak backwardation if the put option value is sufficiently large.

rium production. More specifically, we let the demand shock in the second period become "riskier" in the sense of mean preserving spread (Rothschild and Stiglitz (1971)), and obtain the following result:

Theorem 2 The equilibrium production of oil in at time 0 is non-increasing with the riskiness of the demand shock. That is,
if $\tilde{\epsilon}_1, \tilde{\epsilon}_2 \in \mathcal{E}$ and $\tilde{\xi} \in \mathcal{E}$, where[16] $\tilde{\epsilon}_2 \overset{d}{\sim} \tilde{\epsilon}_1 + \tilde{\xi}$ and $E(\tilde{\xi} \mid \tilde{\epsilon}_1) = 0$, then,

$$q^e(\tilde{\epsilon}_2, \theta) \leq q^e(\tilde{\epsilon}_1, \theta) \ , \tag{14}$$

for all $\theta \in \Theta$.

Proof See Appendix A. □

The result is intuitive — as uncertainty about the second period realization increases, the value of oil in the ground, like the value of a call option, increases or remains unchanged. Hence, with an increase in riskiness of the demand shock no fewer producers will choose to leave their oil in the ground. If uncertainty about future prices becomes extremely high, all producers will choose not to extract and the boundary equilibrium of no production will be reached.
A sufficient condition for positive production at time 0 is presented in appendix A, Theorem A2. More specifically, we show that if the random demand shock is bounded (or alternatively if its variance is bounded) as described in the theorem, positive production at time 0 is guaranteed.

An additional result concerns the sequence of depletion of the oil reserves in the economy.

Theorem 3 In equilibrium, reserves with relatively low extraction cost are exploited before those with high extraction cost; i.e., extraction cost rises as reserves are depleted.

16 " $\overset{d}{\sim}$ " means " is equal in distribution as ".

Proof See Appendix A. □

This result is very intuitive and was mentioned by Hotelling (1931). However, in Hotelling (1931) this outcome is obtained only if extraction cost rises by less than the interest rate, and it results form the interest rate effect. In our economy, in contrast, even if extraction cost rises at the rate of interest the result holds. Here, it is the presence of uncertainty that drives the result. For the lower cost producers the probability of the second period price exceeding their extraction cost is relatively high. Hence, they do not have an incentive to wait for the second period in the presence of backwardation in the market. For the higher cost producers this probability is low, and hence, they would wait for the second period hoping for a favorable demand realization.

3.4 Multiperiod analysis

The above results were obtained in a two period framework. Next, we show that our main result, the existence of weak backwardation for any inner equilibrium, carries over to a multiperiod framework.

Assume a T-period economy with total reserves of oil Q. Further assume a continuum of oil producers who are heterogeneous with respect to their extraction cost. As before, the producers' extraction costs rise over time at the rate of interest and at time T they range from 0 to $\bar{x}$. Each one of the producers can extract the oil at any time throughout time T.

Consider an oil producer with time T extraction cost of x. Denoting by x_t his extraction cost at time t,[17] the producer decides whether or not to produce at time 0 by comparing the net price of oil above ground, $S_0 - x_0$, with its value in the ground. The value of oil in the ground, denoted by $C^T_{\{x_t\}}$, is nothing but a T-period American call option on the spot with an exercise price of $\{x_t\}_{t \leq T}$, increasing over time. In case of an inner equilibrium the time 0 marginal producer is just indifferent between

[17] $x_t \equiv e^{-r(T-t)}x$.

extracting the oil and leaving it in the ground; i.e.,

$$S_0^{e^0} - x_0^{e^0} = C^T_{\{x_t^{e^0}\}} , \tag{15}$$

where $S_0^{e^0}$ is time equilibrium spot price of oil and $x_t^{e^0}$ is the time t extraction cost of the time 0 marginal producer.

The next theorem generalizes the existence of weak backwardation in an inner equilibrium to a multiperiod framework. We make the following notation: $F_t^{e^0}$ denotes the equilibrium t-period futures price as of time 0, $p^t_{x_t^{e^0}}$ denotes the price at time 0 of a t-period European futures put option on with a strike of $x_t^{e^0}$ and finally,

$$B_{w,t}^{e^0} \equiv S_0^{e^0} - e^{-rt} F_t^{e^0} \tag{16}$$

denotes the equilibrium t-period weak backwardation at time 0.[18]

Theorem 4 In an inner equilibrium the market is weakly backwardated with respect any future time t throughout time T. The weak backwardation for time t is no lower than the value of a European put option on futures with an exercise price of $x_t^{e^0}$. That is, for any inner equilibrium we have,

$$B_{w,t}^{e^0} > 0 \quad \text{and} \quad B_{w,t}^{e^0} \geq p^t_{x_t^{e^0}} \quad \text{for all } \ t \leq T . \tag{17}$$

Proof See Appendix A. □

Consider the first result. The zero cost producer will choose to defer extraction whenever $S_0^{e^0} \leq e^{-rt} F_t^{e^0}$ for some $t \leq T$. All other producers, who have higher extraction costs, will choose to defer extraction as well. Hence, a necessary condition

[18]In the two period setting the was no difference between futures and forward prices of oil. However, in the multiperiod framework these prices differ if interest rate is correlated with prices (see Cox, Ingersoll and Ross (1981)). While the analysis refers to forward prices, we continue to use the terminology of futures since the interest rate is assumed to be constant and in order to maintain continuity from previous sections.

for positive production at time 0 is the existence of weak backwardation with respect to each of the future periods. The second result utilizes the the fact that a T-period American call option on the spot is greater in value than any t-period European call option on the futures (for $t \leq T$). This relationship coupled with condition (15) and with put-call parity yields the lower bound for weak backwardation in an inner equilibrium.

The above result is general and does not require the development of a complete multiperiod equilibrium model. We proceed to argue intuitively that increased uncertainty with respect to future prices leads to a decrease in current production. In order to well define this notion of uncertainty we turn to a partial equilibrium analysis. More specifically, we assume that the spot price of oil follows a geometric Brownian Motion with volatility parameter of σ and an unspecified drift. Assuming in addition all other conditions necessary to invoke the Black-Scholes option pricing formula, we can establish the following: Current oil production is decreasing in σ. This result is clear when we consider the time 0 marginal producer. Other things equal, an increase in the price volatility will lead this producer to decrease production since the value of the oil well (an American call option) is decreasing in σ. Hence, current oil production is decreasing in σ.

4 Empirical Evidence

In this section we test some empirical implications of the model developed above. The empirical evidence is consistent with the model. Non-regulated oil production within the U.S. is found to be positively related to the level of weak backwardation and negatively related to implied volatility. Also, weak backwardation is shown to have a significant positive relationship with the at-the-money put option price and with implied volatility.

4.1 Production, backwardation and volatility

In the previous section it was shown that higher spot price of oil leads to an increase in current supply, whereas higher call option prices strengthen the incentive to defer production. We wish to examine these predictions empirically. We note that in practice, changing oil prices affect the intensity of field development and exploration. These, in turn, lead to fluctuations in extraction costs and in reserves. Therefore, it is more appropriate to use relative variables for the test as opposed to absolute variables. Production rate (production as ratio of reserves) is taken to be the dependent variable. Weak backwardation rate (relative price of the spot versus the futures) and implied volatility are taken to be the explanatory variables.[19] Since producers increase extraction as the spot price appreciates relative to the futures price, we expect a positive relationship between production rate and backwardation rate. We expect to find a negative relationship between production rate and volatility since higher volatility induces producers to defer extraction (as a result of the option effect).

The United States oil production and reserves data, as well as the WTI crude oil futures and option prices, are used to construct the aforementioned variables.[20] Since

[19]Exact definitions of the variables follow.

[20]The monthly data on production and reserves was obtained from the Energy Information Administration (the Department of Energy). The futures and options data are for the WTI (West Texas Intermediate) crude oil contracts listed on the New York Mercantile Exchange.

some of the states regulate production and others do not, we measure production rate separately for each of the two groups — the regulated and the non-regulated states. The backwardation is measured for the second, third and fourth nearby contracts relative to the first nearby contract, where the latter serves as a proxy for the spot price.[21,22] The implied volatility is obtained from the at-the-money call option on the corresponding nearby futures contract, using the Black (1976) formula. The longest horizon examined is four months since the options are not actively traded beyond the fourth nearby contract.

Production rate (monthly production divided by reserves), q, is regressed on lagged monthly averaged weak backwardation rate, $\bar{BR}^w_\tau$, and on lagged monthly averaged implied volatility, $\bar{\sigma}_\tau$:

$$q_t = \alpha_\tau + \beta_\tau \bar{BR}^w_{\tau,t-1} + \gamma_\tau \bar{\sigma}_{\tau,t-1} + \epsilon_{\tau,t} \ . \tag{18}$$

Weak backwardation rate is defined to be the BR^w_τ that satisfies $S = e^{-(r_\tau - BR^w_\tau)\tau} F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity. The implied volatility, σ_τ , is computed from the τ-th month at-the-money call option price using the Black (1976) formula. The sample is for the period December 1986 through December 1991.[23]

The results are presented in Table 2. The t-statistics refer to the null hypothesis that the corresponding coefficients are zero, whereas the χ^2-statistic refers to the null that

[21] The first nearby futures contract is the nearest to maturity contract and its time to expiration ranges from one day to thirty one days. The second nearby contract has one additional month to expiration. The third nearby has two additional months, etc.

[22] There is no organized trading in standardized spot contracts for oil and therefore quoted spot prices will vary by dealer, quality and place of delivery. To this extent, the use of the first nearby futures price is preferable.

[23] While futures prices and aggregate production rates are available earlier, data on option prices are available from December 1986. To make use of the longer data for futures prices and production all tests were conducted using time series estimated volatilities. The results obtained were similar.

all slope coefficients are zero. The standard errors used to calculate these statistics were computed using Hansen's (1982) covariance matrix.[24] Consistent with the underlying theory, the coefficients on the backwardation are positive, although non-significant, and the coefficients on the volatility are significantly negative.[25] Though not presented, we note that the absolute value of the coefficients as well as the level of significance are lower for the regulated states than for the non-regulated states.

4.2 Backwardation, put option price and volatility

As suggested by Theorem 1 in case of an inner equilibrium weak backwardation is equal to the value of a put option on the second period futures price with a strike equaling the extraction cost of the marginal producer. Theorem 4 provides a lower bound for weak backwardation in a multiperiod framework — weak backwardation is no lower than the value of a European futures put option with a strike equaling the time t extraction of the marginal producer. We wish to examine empirically the relationship between backwardation and the put option values, expecting a significant positive relationship. However, the extraction cost of the marginal producer at each point in time is unknown. Therefore, at-the-money put options are being used (discussion follows). An alternative approach is to use the implied volatility from traded options as an explanatory variable for backwardation.

Weak backwardation for the second, third and fourth nearby futures contracts, B^w_τ, is regressed on the corresponding at-the-money put option price, P_τ , over the period December 1986 through December 1991:

$$B^w_{\tau,t} = \alpha_\tau + \beta_\tau P_{\tau,t} + \epsilon_{\tau,t} \ . \tag{19}$$

Weak backwardation is defined by $B^w_\tau = S - e^{-r_\tau \tau} F_\tau$, where S is the price of the

[24]case (iii) with truncation after two lags.

[25]Similar results were obtained for the regression of production rate on the nearest to maturity futures price (as proxy for the spot) and call option prices. The coefficient was non-significantly positive for the former and significantly negative for the latter.

nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity. The results are presented in Table 3. The t-statistics refer to the null hypothesis that the corresponding coefficients are zero.[26] Consistent with our conjecture, there is a highly significant positive correlation between backwardation and the at-the-money put price.

Considering the absolute magnitude of the put price coefficients, we note that they are less than one and monotonically increase with maturity. As the put options used in the regression differ from those suggested by the model, two possible effects are considered. On the one hand, the short term American put options used are less valuable than long term American put options associated with the long term nature of oil wells. This would suggest higher than one slope coefficients. On the other hand, the put options used are at-the-money. To the extent that the extraction cost of the marginal producer is lower than the prevailing futures price, these options are more valuable than those suggested by the model. Furthermore, since the market is backwardated (futures prices decrease with maturity) and the extraction cost rises over time, the gap in value between put options used in the regression and those suggested by the model should decrease with maturity. This would suggest smaller than one slope coefficients which increase with maturity, as observed in table 3.

As explained above, the put option suggested by the model are not observable. Nevertheless, these options as well as the observed options are positively related to the expected volatility of oil prices. Hence, an alternative approach is to use the implied volatility from traded options as an explanatory variable for backwardation. We regress weak backwardation for the second, third and fourth nearby futures contract, B^w_τ, on

[26] Once again the standard errors used to calculate these statistics were computed using Hansen's (1982) covariance matrix (case (iii) with truncation after two lags).

the implied volatility from the corresponding at-the-money call option price, σ_T:[27]

$$B^w_{T,t} = \alpha_T + \beta_T \sigma_{T,t} + \epsilon_{T,t} \ . \tag{20}$$

Weak backwardation and implied volatility are defined above. The sample is unchanged as well. As can be seen in Table 4, the relation is positive and highly significant and it provides further reinforcement for the previous result.

The above results suggest that the preponderance of backwardation in the oil market results from rational extraction decisions of producers. This raises the question concerning whether the small percentage of observations when the oil market was not in weak backwardation is prima facia evidence of irrational behavior. In section 3.3 we briefly mentioned that many oil wells, such as stripper wells, must have a minimum level of production at any given time to be able to continue production in the future. If these high cost producers cease production at present, future production would be reduced. When the price of oil is low and marginal producers are losing money at time 0 in order to maintain their options to produce in the future, it is possible for the oil market to be in contango (spot price lower than the discounted futures price). Figure 2 provides support for this conjecture. As can be seen in the figure, periods of negative backwardation are associated with relatively low price for oil.

[27] In order not to use the same put options data to estimate the implied volatilities, we use call options for this purpose.

5 Conclusions

This paper presents a theory of backwardation in oil futures markets. The paper focuses on the producer's behavior and demonstrates that the call option feature of oil in the ground is the source of backwardation.

In a two period framework it is shown that the market is always weakly backwardated in an inner equilibrium. The weak backwardation is equal to the value of a put option with an exercise price equaling the extraction cost of the marginal producer. If the value of the put is sufficiently large strong backwardation emerges. Equilibrium production is shown to be non-increasing in the riskiness of future prices. The existence of weak backwardation in an inner equilibrium is generalized to a multiperiod framework — the market is weakly backwardated with respect to any future time t. Weak backwardation is bounded from below by the value of a t-period European futures put option with a strike equal to the time t extraction cost of the marginal producer.

The empirical evidence is consistent with the predictions of the model. Non-regulated oil production within the U.S. is found to be positively related to the level of weak backwardation and negatively related to the implied volatility. Also, weak backwardation is shown to have a significant positive relationship with the at-the-money put option price and with implied volatility.

Appendix A

Define the function $H(x(\tilde{\epsilon},\theta);\tilde{\epsilon},\theta)$ as follows (suppressing the arguments of x):

$$\begin{aligned} H(x;\tilde{\epsilon},\theta) &\equiv S_0 - e^{-r}x - C(\tilde{S}_1,x,r) \\ &= \frac{a}{b} - \frac{Q_0}{b\bar{x}}x - e^{-r}x - e^{-r}E(\tilde{S}_1 - x \mid \tilde{S}_1 \geq x)P(\tilde{S}_1 \geq x) \ . \end{aligned} \tag{21}$$

Also define the following functions ($i = 1,\ 2$):

$$c_i(\tilde{\epsilon},k_i,\theta) \equiv e^{-r}E(\tilde{\epsilon} - k_i \mid \tilde{\epsilon} \geq k_i)P(\tilde{\epsilon} \geq k) \ , \tag{22}$$

where

$$k_1 = bx - a + Q_1\frac{x}{\bar{x}} \tag{23}$$

$$k_2 = b\bar{x} - a + Q - Q_0\frac{x}{\bar{x}} \ . \tag{24}$$

Note that the function $c_i(\tilde{\epsilon},k_i,\theta)$ is the value of an option written on ϵ with the the exercise price of k_i .

The option $C(\tilde{S}_1,x,r)$ pays off at time 1 $\max[\tilde{S}_1 - x, 0]$. It is easy to verify that,

$$\max[\tilde{S}_1 - x, 0] \equiv \frac{\bar{x}}{b\bar{x}+Q}\max[\tilde{\epsilon} - k_1, 0] + \frac{Q}{b(b\bar{x}+Q)}\max[\tilde{\epsilon} - k_2, 0] \ , \tag{25}$$

which implies that

$$C(\tilde{S}_1,x,r) \equiv \frac{\bar{x}}{b\bar{x}+Q}c_1(\tilde{\epsilon},k_1,\theta) + \frac{Q}{b(b\bar{x}+Q)}c_2(\tilde{\epsilon},k_2,\theta) \ . \tag{26}$$

Hence, the function $H(x;\tilde{\epsilon},\theta)$ can be written as:

$$H(x;\tilde{\epsilon},\theta) = \frac{a}{b} - \frac{Q_0}{b\bar{x}}x - e^{-r}x - \frac{\bar{x}}{b\bar{x}+Q}c_1(\tilde{\epsilon},k_1,\theta) - \frac{Q}{b(b\bar{x}+Q)}c_2(\tilde{\epsilon},k_2,\theta) \ . \tag{27}$$

We establish now the following results which are used in the proofs to follow:

Lemma 1 $H(x;\tilde{\epsilon},\theta)$ is decreasing in x ; i.e.,

$$\frac{\partial H(x;\tilde{\epsilon},\theta)}{\partial x} < 0 \ . \tag{28}$$

Proof of lemma 1 By the standard option result that $-e^{-r} \leq \frac{\partial c_i}{\partial k_i} \leq 0$ we obtain from (27)

$$\begin{aligned} \frac{\partial H}{\partial x} &= -\frac{Q_0}{b\bar{x}} - e^{-r} - \frac{b\bar{x}+Q_1/\bar{x}}{b\bar{x}+Q}\frac{\partial c_1}{\partial k_1} + \frac{QQ_0}{b\bar{x}(b\bar{x}+Q)}\frac{\partial c_2}{\partial k_2} \\ &\leq -\frac{Q_0}{b\bar{x}} - e^{-r} - \frac{b\bar{x}+Q_1/\bar{x}}{b\bar{x}+Q}\frac{\partial c_1}{\partial k_1} \\ &\leq -\frac{Q_0}{b\bar{x}} - \frac{e^{-r}Q_0}{\bar{x}+Q} \\ &< 0 \ . \end{aligned}$$

□

Lemma 1a Let $J(x) = S_0 - e^{-r}x - C(\tilde{s}_1, x, 0)$. Then, $J(x)$ is decreasing in x ; i.e.,

$$\frac{\partial J(x)}{\partial x} < 0 .$$

Proof of lemma 1a By the standard option result that $-e^{-r} \leq \frac{\partial C}{\partial x} \leq 0$ we obtain:

$$\frac{\partial J}{\partial x} = -e^{-r} - \frac{\partial C}{\partial x} \leq 0 .$$

□

Lemma 2 Let $\tilde{\epsilon}_1, \tilde{\epsilon}_2 \in \mathcal{E}$ and $\tilde{\xi} \in \mathcal{E}$, where [28] $\tilde{\epsilon}_2 \overset{d}{\sim} \tilde{\epsilon}_1 + \tilde{\xi}$ and $E(\tilde{\xi} \mid \tilde{\epsilon}_1) = 0$, Then,

$$H(x; \tilde{\epsilon}_1, \theta) \geq H(x; \tilde{\epsilon}_2, \theta) , \tag{29}$$

for all $x \in [0, \bar{x}]$ and for all $\theta \in \Theta$.

Proof of lemma 2 The value of a European call option is non-decreasing in increased riskiness in the sense of mean preserving spread (See Merton (1973)). Thus, we have

$$c_i(\tilde{\epsilon}_1, k, \theta) \leq c_i(\tilde{\epsilon}_2, k, \theta) .$$

The desired result follows immediately from the definition of $H(x, \tilde{\epsilon}, \theta)$. □

Theorem A1 Existence and uniqueness of equilibrium (necessary and sufficient conditions):
Let $q^e(\tilde{\epsilon}, \theta)$ be the equilibrium production.
Then,

1. For all $(\tilde{\epsilon}, \theta) \in (\mathcal{E} \times \Theta)$ such that

$$\frac{a}{b} \leq e^{-r} F^e , \tag{30}$$

   we have $q^e(\tilde{\epsilon}, \theta) = 0$.

2. For all $(\tilde{\epsilon}, \theta) \in (\mathcal{E} \times \Theta)$ such that

$$\frac{a - Q_0}{b} - e^{-r}\bar{x} \geq C(\tilde{S}_1, \bar{x}, r) , \tag{31}$$

   we have $q^e(\tilde{\epsilon}, \theta) = Q_0$.

[28] " $\overset{d}{\sim}$ " means " is equal in distribution as ".

3. For all $(\tilde{\epsilon}, \theta) \in (\mathcal{E} \times \Theta)$ such that

$$\frac{a}{b} > e^{-r} F^e , \tag{32}$$

$$\text{and} \quad \frac{a - Q_0}{b} - e^{-r}\bar{x} < C(\tilde{S}_1, \bar{x}, r) , \tag{33}$$

there exists a unique $q^e(\tilde{\epsilon}, \theta) \in (0, Q_0)$.

Proof of theorem A1

1. Necessity:
$q^e(\tilde{\epsilon}, \theta) = 0$ implies $H(0; \tilde{\epsilon}, \theta) \leq 0$. Hence,

$$H(0; \tilde{\epsilon}, \theta) = \frac{a}{b} - C(\tilde{S}_1, 0, r) \leq 0 .$$

The result (equation (30)) follows immediately since $e^{-r}F^e = C(\tilde{S}_1, 0, r)$.

Sufficiency:
Let $a/b \leq e^{-r}F^e$ and suppose that $q^e(\tilde{\epsilon}, \theta) > 0$. It follows that (using lemma 1 for the strict inequality),

$$\begin{aligned} 0 &\leq H(x^e; \tilde{\epsilon}, \theta) \\ &< H(0; \tilde{\epsilon}, \theta) \\ &= \frac{a}{b} - e^{-r}F^e \\ &< 0 . \quad \rightarrow\leftarrow \end{aligned}$$

Hence, $q^e(\tilde{\epsilon}, \theta) = 0$.

2. Necessity:
$q^e(\tilde{\epsilon}, \theta) = Q_)$ implies $H(\bar{x}; \tilde{\epsilon}, \theta) \geq 0$. Hence,

$$H(\bar{x}; \tilde{\epsilon}, \theta) = \frac{a - Q_0}{b} - e^{-r}\bar{x} - C(\tilde{S}_1, \bar{x}, r) \geq 0 .$$

The result (equation (31)) follows immediately.

Sufficiency:
Let $\frac{a-Q}{b} - e^{-r}\bar{x} \geq C(\tilde{S}_1, \bar{x}, r)$ and suppose that $q^e(\tilde{\epsilon}, \theta) < Q_0$. It follows that (using lemma 1 for the strict inequality),

$$\begin{aligned} 0 &\geq H(x^e; \tilde{\epsilon}, \theta) \\ &> H(\bar{x}; \tilde{\epsilon}, \theta) \\ &= \frac{a - Q_0}{b} - e^{-r}\bar{x} - C(\tilde{S}, \bar{x}, r) \\ &\geq 0 . \quad \rightarrow\leftarrow \end{aligned}$$

Hence, $q^e(\tilde{\epsilon}, \theta) = Q_0$.

3. To prove the necessity and sufficiency of the conditions for the existence of an inner equilibrium (equations (32) and (33)), as well as its uniqueness, just combine the first and second results in this theorem together with lemma 1. The third result follows then by the intermediate value theorem. □

Theorem A2 Sufficient conditions for inner equilibrium :

1. If $(\tilde{\epsilon}, \theta) \in (\mathcal{E} \times \Theta)$ and

$$Var(\tilde{\epsilon}) < Q \ , \tag{34}$$

$$\text{and} \quad \frac{a - Q_0}{b} \leq e^{-r}\bar{x} \ , \tag{35}$$

then $q^e(\tilde{\epsilon}, \theta) \in (0, Q_0)$.

2. If $(\tilde{\epsilon}, \theta) \in (\mathcal{E} \times \Theta)$ and

$$\tilde{\epsilon} < Q, \tag{36}$$

$$\text{and} \quad \frac{a - Q_0}{b} \leq e^{-r}\bar{x} \ , \tag{37}$$

then $q^e(\tilde{\epsilon}, \theta) \in (0, Q_0)$.

Proof of theorem A2

1. (a) If $Var(\tilde{\epsilon}) < Q$, then

$$\begin{aligned}
& H(0; \tilde{\epsilon}, \theta) = \\
= \ & \frac{a}{b} - \frac{\bar{x}}{b\bar{x} + Q} c_1(\tilde{\epsilon}, -a, \theta) - \frac{Q}{b(b\bar{x} + Q)} c_2(\tilde{\epsilon} b\bar{x} - a + Q, \theta) & (38) \\
= \ & \frac{a}{b} - \frac{e^{-r}\bar{x}}{b\bar{x} + Q} E(\tilde{\epsilon} \mid \tilde{\epsilon} \geq -a) P(\tilde{\epsilon} \geq -a) + \frac{e^{-r}\bar{x}a}{b\bar{x} + Q} P(\tilde{\epsilon} \geq -a) \\
- \ & \frac{e^{-r}Q}{b(b\bar{x} + Q)} E(\tilde{\epsilon} \mid \tilde{\epsilon} \geq b\bar{x} - a + Q) P(\tilde{\epsilon} \geq b\bar{x} - a + Q) \\
+ \ & \frac{e^{-r}Q(b\bar{x} - a + Q)}{b(b\bar{x} + Q)} P(\tilde{\epsilon} \geq b\bar{x} - a + Q) & (39) \\
\geq \ & e^{-r}\frac{Q}{b} + \frac{a}{b}(1 - e^{-r}) \\
- \ & \frac{e^{-r}\bar{x}}{b\bar{x} + Q} E(\tilde{\epsilon} \ 1_{\{\tilde{\epsilon} \geq -a\}}) - \frac{e^{-r}Q}{b(b\bar{x} + Q)} E(\tilde{\epsilon} \ 1_{\{\tilde{\epsilon} \geq b\bar{x} - a + Q\}}) & (40) \\
\geq \ & e^{-r}\frac{Q}{b} - \frac{e^{-r}\bar{x}}{b\bar{x} + Q} E|\tilde{\epsilon} \ 1_{\{\tilde{\epsilon} \geq -a\}}| - \frac{e^{-r}Q}{b(b\bar{x} + Q)} E|\tilde{\epsilon} \ 1_{\{\tilde{\epsilon} \geq b\bar{x} - a + Q\}}| & (41) \\
\geq \ & e^{-r}\frac{Q}{b} - \frac{e^{-r}\bar{x}}{b\bar{x} + Q} (E \ |\tilde{\epsilon}|^2)^{\frac{1}{2}} (E \ |1_{\{\tilde{\epsilon} \geq -a\}}|^2)^{\frac{1}{2}} \\
- \ & \frac{e^{-r}Q}{b(b\bar{x} + Q)} (E \ |\tilde{\epsilon}|^2)^{\frac{1}{2}} (E \ |1_{\{\tilde{\epsilon} \geq b\bar{x} - a + Q\}}|^2)^{\frac{1}{2}} & (42) \\
\geq \ & e^{-r}\frac{Q}{b} - e^{-r}\frac{1}{b}(E \mid \tilde{\epsilon} \mid^2)^{\frac{1}{2}} & (43) \\
> \ & 0 \ .
\end{aligned}$$

To pass from (39) to (40) we note that the probability takes no values larger than 1. The passage from (40) to (41) is due to the introduction of absolute values. We used the Cauchy-Schwartz inequality and the sufficient condition to pass from (41) to (42). In passing from (42) to (43) we utilize the boundedness of the indicator function by 1 and then collect terms. The last inequality is obtained from the sufficient condition. Hence, if $Var(\tilde{\epsilon}) < Q$ then there is positive production at time 0.

(b)

$$
\begin{aligned}
& H(\bar{x};\tilde{\epsilon},\theta) = \\
= \; & \frac{a - Q_0}{b} - e^{-r}\bar{x} \\
- \; & e^{-r}\frac{1}{b}E(\tilde{\epsilon} - b\bar{x} + a + Q_1 \mid \tilde{\epsilon} - b\bar{x} + a + Q_1 \geq 0)P(\tilde{\epsilon} - b\bar{x} + a + Q_1 \geq 0) \\
\leq \; & \frac{a - Q_0}{b} - e^{-r}\bar{x} \\
< \; & 0\,.
\end{aligned}
$$

The first inequality holds since the second term is non-negative A sufficient condition for the second inequality is $\frac{a-Q}{b} \leq e^{-r}\bar{x}$. Hence, if this is the case, some reserves will be left for the second period .

2. (a) Assume that $\tilde{\epsilon}$ is bounded by L. Then,

$$
\begin{aligned}
& H(0;\tilde{\epsilon},\theta) = \\
= \; & \frac{a}{b} - \frac{e^{-r}\bar{x}}{b\bar{x}+Q}E(\tilde{\epsilon} + a \mid \tilde{\epsilon} \geq -a)P(\tilde{\epsilon} \geq -a) \\
- \; & \frac{e^{-r}Q}{b(b\bar{x}+Q)}E(\tilde{\epsilon} - b\bar{x} + a - Q \mid \tilde{\epsilon} \geq b\bar{x} - a + Q)P(\tilde{\epsilon} \geq b\bar{x} - a + Q) \\
\geq \; & \frac{a}{b} - \frac{e^{-r}\bar{x}(L+a)}{b\bar{x}+Q} - \frac{e^{-r}(L - b\bar{x} + a - Q)}{b(b\bar{x}+Q)} \\
\geq \; & e^{-r}\frac{Q-L}{b} \\
> \; & 0\,.
\end{aligned}
$$

The last inequality holds if $Q > L$. Hence, $\tilde{\epsilon}$ bounded by Q is a sufficient condition for positive production at time 0.

(b) Same as 1(b) above.

□

Proof of theorem 2 The proof is given for the case of an inner equilibrium; the reasoning can be exactly reproduced for the boundary equilibria. Let $\tilde{\epsilon}_1, \tilde{\epsilon}_2 \in \mathcal{E}_{\setminus\{0\}}$ and $\tilde{\xi} \in \mathcal{E}_{\setminus\{0\}}$, where $\tilde{\epsilon}_2 \sim \tilde{\epsilon}_1 + \tilde{\xi}$ and $E(\tilde{\xi} \mid \tilde{\epsilon}_1) = 0$. Suppose that for all $\theta \in \Theta$,

$$q^e(\tilde{\epsilon}_2,\theta) > q^e(\tilde{\epsilon}_1,\theta)$$

This in turn implies that,

$$x_2^e \equiv x^e(\tilde{\epsilon}_2,\theta) > x^e(\tilde{\epsilon}_1,\theta) \equiv x_1^e \ .$$

Now, using lemma 1 for the first inequality and lemma 2 for the second inequality, we obtain,

$$\begin{aligned} 0 &= H(x_1^e,\tilde{\epsilon}_1,\theta) \\ &> H(x_2^e,\tilde{\epsilon}_1,\theta) \\ &\geq H(x_2^e,\tilde{\epsilon}_2,\theta) \\ &= 0 \ . \quad \rightarrow\leftarrow \end{aligned}$$

Hence, $q^e(\tilde{\epsilon}_2,\theta) \leq q^e(\tilde{\epsilon}_1,\theta)$. □

Proof of theorem 3 From lemma 1 $H(x;\tilde{\epsilon},\theta)$ is decreasing in x . Thus, any producer with extraction cost x that is higher than $x^e(\tilde{\epsilon},\theta)$ will choose not to extract at time 0 since

$$H(x;\tilde{\epsilon},\theta) < H(x^e;\tilde{\epsilon},\theta) \leq 0 \ .$$

Similarly it can be shown that producers with extraction cost x that is lower than $x^e(\tilde{\epsilon},\theta)$ will choose to extract their oil at time 0. Hence, the result is obtained. □

Proof of theorem 4

1. The zero cost producer will choose to defer extraction whenever $S_0^{e^0} \leq e^{-rt}F_t^{e^0}$ for some $t \leq T$. All other producers, who have higher extraction costs, will choose to defer extraction as well. Hence, a necessary condition for positive production at time 0 is $B_{w,t}^{e^0} > 0$ for all $t \leq T$.

2. $C^T_{\{x_u^{e^0}\}}$ is a T-period American call option on the spot with a strike of $\{x_u^{e^0}\}_{u\leq T}$ where $x_u^{e^0} = e^{-r(T-u)}x^{e^0}$. It is no lower in value than a t-period American call on the spot with a strike of $\{x_u^{e^0}\}_{u\leq t}$. This, in turn, is no lower in value than a t-period European call option on the spot with a strike $x_t^{e^0}$, which equals a t-period call option on the futures with the same strike. Denoting the latter by $c^t_{x_t^{e^0}}$, the above inequalities can be summarized as

$$C^T_{\{x_u^{e^0}\}} \geq c^t_{x_t^{e^0}} \quad \text{for all} \ \ t \leq T \ . \tag{44}$$

Put call parity implies

$$c^t_{x_t^{e^0}} = p^t_{x_t^{e^0}} + e^{-rt}(F_t^{e^0} - x_t^{e^0}) \ . \tag{45}$$

Now combine the equilibrium condition (15) with (44) and (45) to obtain $B_{w,t}^{e^0} \geq p^t_{x_t^{e^0}}$ for all $t \leq T$.

□

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Table 1: **Descriptive Statistics for Backwardation**

The descriptive statistics were computed for the period of February 1984 through April 1992. Weak backwardation is defined by $B_\tau^w = S - e^{-r_\tau \tau} F_\tau$, and strong backwardation is defined by $B_\tau^s = S - F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity.

Panel A: Weak Backwardation (in dollars)

Futures contract	2nd	3rd	4th	5th	6th	7th	8th	9th
Average	0.29	0.49	0.64	0.76	0.87	0.96	1.04	1.11
Median	0.21	0.35	0.44	0.53	0.59	0.64	0.68	0.71
S. Deviation	0.48	0.79	1.04	1.25	1.43	1.57	1.70	1.82
Minimum	-2.05	-2.14	-2.67	-3.04	-3.34	-3.52	-3.66	-3.76
Maximum	3.53	4.70	5.79	6.82	7.80	8.70	9.82	10.80

Panel B: Strong Backwardation (in dollars)

Futures contract	2nd	3rd	4th	5th	6th	7th	8th	9th
Average	0.24	0.43	0.58	0.70	0.81	0.90	0.98	1.05
Median	0.15	0.29	0.38	0.47	0.53	0.58	0.61	0.65
S. Deviation	0.48	0.78	1.03	1.24	1.42	1.57	1.70	1.82
Minimum	-2.09	-2.19	-2.72	-3.09	-3.39	-3.58	-3.72	-3.82
Maximum	3.43	4.60	5.70	6.75	7.71	8.61	9.73	10.70

Panel C: Fraction of the Time in Backwardation (in %)

Futures contract	2nd	3rd	4th	5th	6th	7th	8th	9th
Weak	81.80	85.08	87.04	89.38	90.31	91.63	93.00	93.88
Strong	71.09	71.87	72.41	73.09	74.51	75.00	76.13	76.91

Table 2: **Production, Backwardation and Volatility**

$$q_t = \alpha_\tau + \beta_\tau \bar{BR}^w_{\tau,t-1} + \gamma_\tau \bar{\sigma}_{\tau,t-1} + \epsilon_{\tau,t}$$

Production rate (monthly production divided by reserves), q, is regressed on lagged monthly averaged weak backwardation rate, $\bar{BR}^w_\tau$, and on lagged monthly averaged implied volatility, $\bar{\sigma}_\tau$. Weak backwardation rate is defined to be the BR^w_τ that satisfies $S = e^{-(r_\tau - BR^w_\tau)\tau} F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity. The implied volatility, σ_τ , is computed from the τ-th month at-the-money call option price using the Black (1976) formula. All variables are expressed in percentage annual terms. The t-statistics refer to the null hypothesis that the corresponding coefficient is zero, whereas the χ^2-statistic refers to the null that all slope coefficients are zero. The standard errors used to calculate these statistics were computed using Hansen's (1982) covariance matrix.

December 1986 - December 1991 (60 obs.)

	$\tau = 2$			$\tau = 3$			$\tau = 4$		
	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\gamma}_2$	$\hat{\alpha}_3$	$\hat{\beta}_3$	$\hat{\gamma}_3$	$\hat{\alpha}_4$	$\hat{\beta}_4$	$\hat{\gamma}_4$
Estimator	9.349	0.082	-0.025	9.343	0.085	-0.026	9.342	0.059	-0.026
t-statistic	24.642	0.549	-4.044	23.735	0.401	-3.092	22.429	0.219	-2.453
p-value	0.000	0.293	0.000	0.000	0.345	0.002	0.000	0.414	0.009
χ^2-statistic	16.953			11.133			7.572		
p-value	0.000			0.004			0.023		

Table 3: **Backwardation and the Put Option Price**

$$B^w_{\tau,t} = \alpha_\tau + \beta_\tau P_{\tau,t} + \epsilon_{\tau,t}$$

Weak backwardation for the τ-th month futures contract, B^w_τ, is regressed on the corresponding at-the-money put option price, P_τ. Weak backwardation is defined by $B^w_\tau = S - e^{-r_\tau \tau} F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity. The t-statistics refer to the null hypothesis that the corresponding coefficient is zero. The standard errors used to calculate these statistics were computed using Hansen's (1982) covariance matrix.

December 1986 - April 1992 (1222 obs.)

	$\tau = 2$		$\tau = 3$		$\tau = 4$	
	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\alpha}_3$	$\hat{\beta}_3$	$\hat{\alpha}_4$	$\hat{\beta}_4$
Estimator	0.047	0.250	-0.091	0.557	-0.278	0.831
t-statistic	1.168	8.312	-1.401	15.370	-3.028	15.471
p-value	0.121	0.000	0.080	0.000	0.001	0.000

Table 4: **Backwardation and Volatility**

$$B^w_{\tau,t} = \alpha_\tau + \beta_\tau \sigma_{\tau,t} + \epsilon_{\tau,t}$$

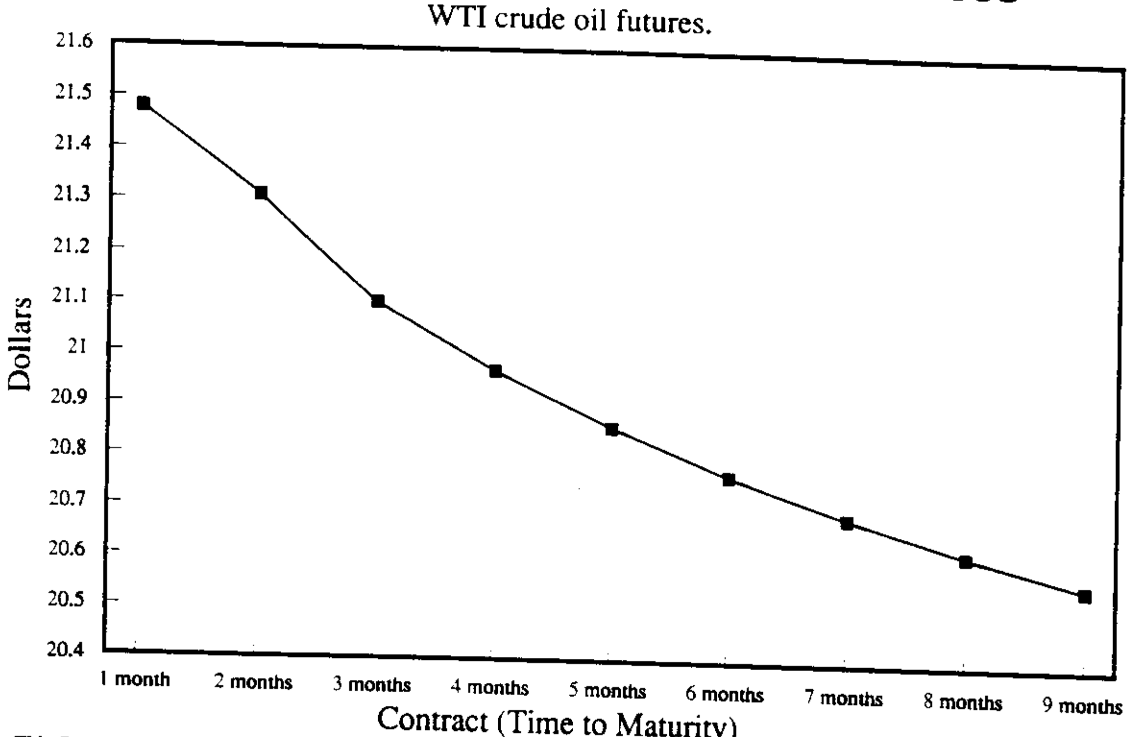
Weak backwardation for the τ-th month futures contract, B^w_τ, is regressed on the implied volatility from the corresponding at-the-money call option price, σ_τ. Weak backwardation is defined by $B^w_\tau = S - e^{-r_\tau \tau} F_\tau$, where S is the price of the nearest to maturity contract, F_τ is the price of the τ-th nearby futures contract and r_τ is the LIBOR rate for the corresponding maturity. The implied volatility, σ_τ , (in percentage annual terms) is computed using the Black (1976) formula. The t-statistics refer to the null hypothesis that the corresponding coefficient is zero. The standard errors used to calculate these statistics were computed using Hansen's (1982) covariance matrix.

December 1986 - April 1992 (1222 obs.)

	$\tau = 2$		$\tau = 3$		$\tau = 4$	
	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\alpha}_3$	$\hat{\beta}_3$	$\hat{\alpha}_4$	$\hat{\beta}_4$
Estimator	-0.033	0.011	-0.287	0.028	-0.631	0.048
t-statistic	-0.601	6.858	-2.838	8.787	-4.112	9.213
p-value	0.274	0.000	0.002	0.000	0.000	0.000

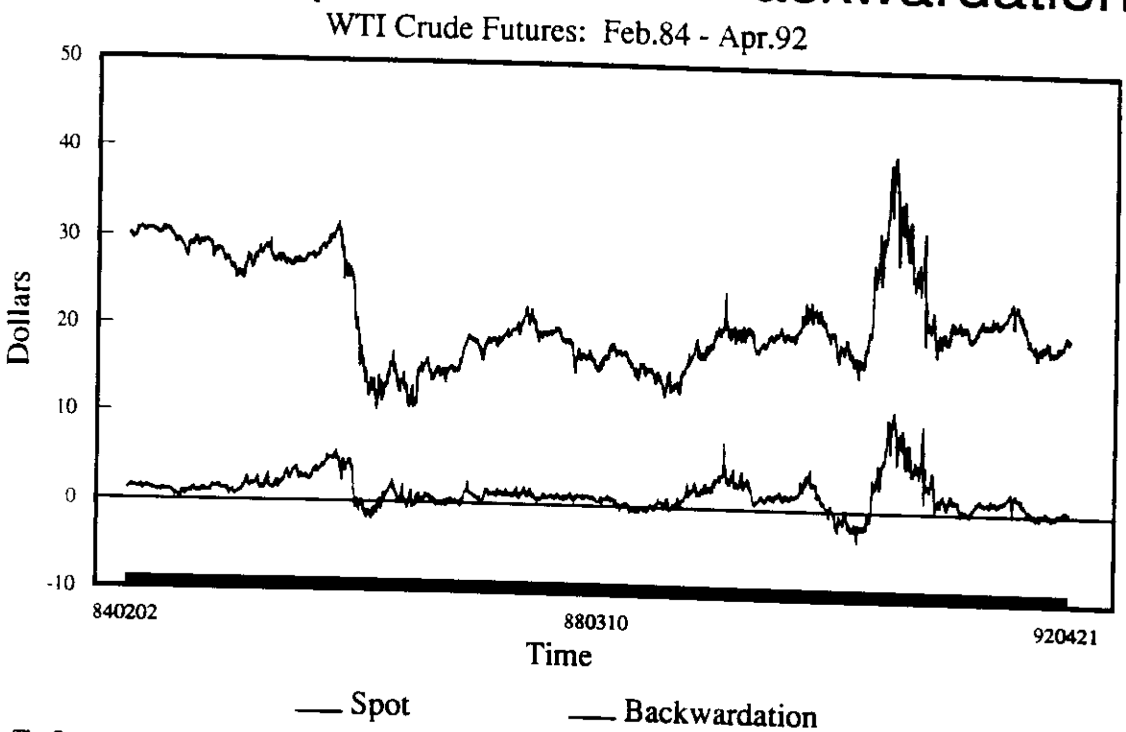
Figure 1: Term Structure of Futures Prices

WTI crude oil futures.



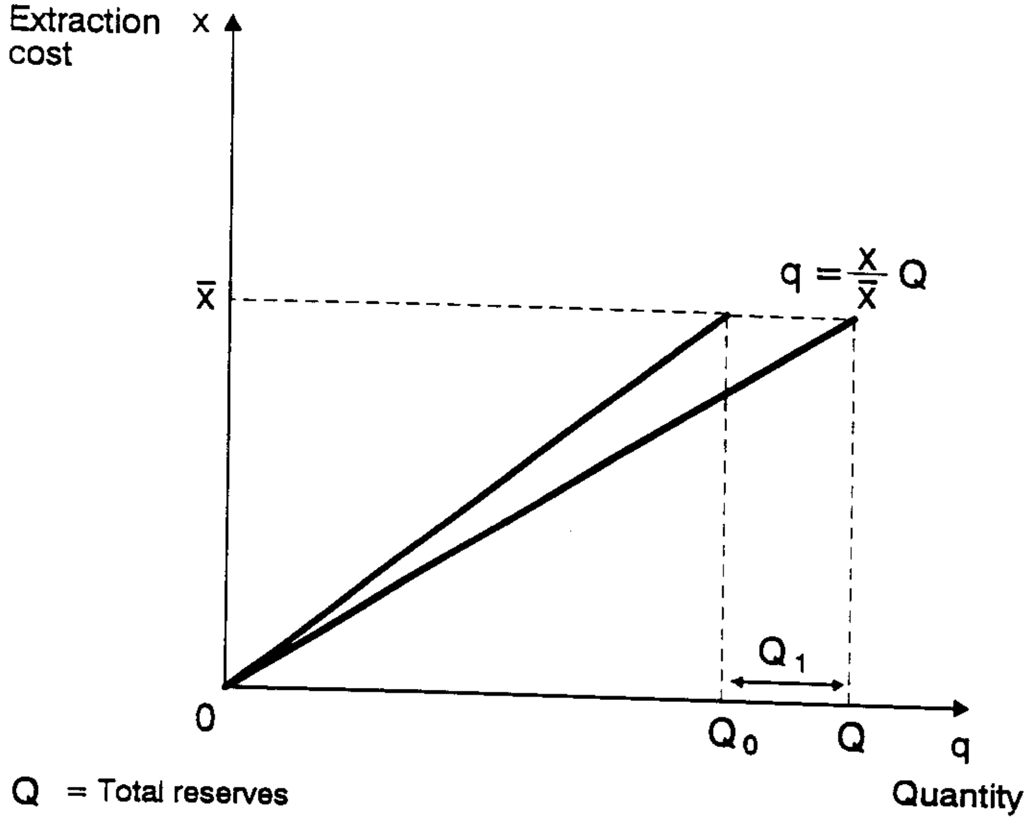
This figure presents the term structure of futures prices for the WTI contracts (traded on the NYMEX) on a typical day of strong backwardation (11/29/91).

Figure 2: Spot Price and Backwardation



The figure presents the futures price of the nearest to maturity contract (which is a proxy for the spot price) and the weak backwardation of the ninth nearby futures price vs. the nearest to maturity futures price.

Figure 3 : Production Technology in the Economy

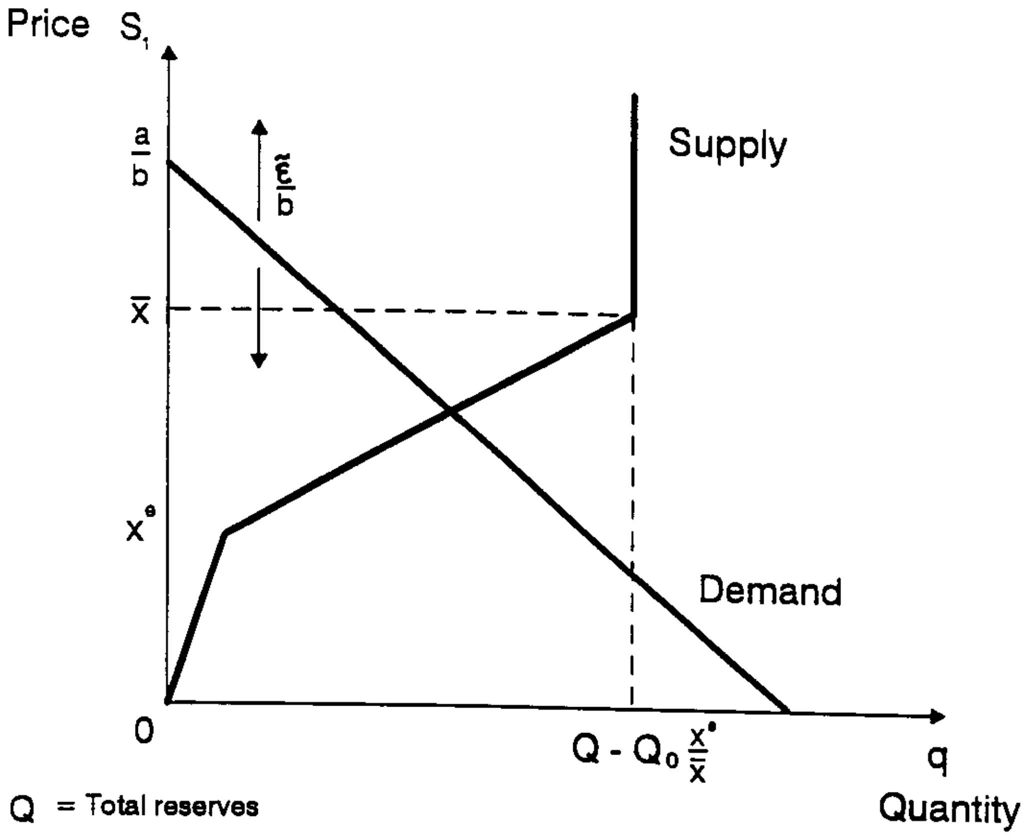


Q = Total reserves

Q_0 = Time 0 available reserves

$\bar{x}$ = Highest extraction cost

Figure 4 : Supply and Demand at time 1



Q = Total reserves

Q_0 = Time 0 available reserves

$\bar{X}$ = Highest extraction cost

X^e = Marginal producer's extraction cost